WEAK PRODUCTS, HANKEL OPERATORS, AND INVARIANT SUBSPACES

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When studying Hardy and Bergman spaces of analytic functions on a region, it is natural to view them as part of the family of H^p- or L^p_a-spaces, and investigate how properties of the functions and operators on these spaces change as the parameter p changes. For reproducing kernel Hilbert spaces like the Dirichlet space of the unit disc or the Drury-Arveson space of the unit ball of $\mathbb{C}^d$ it is unclear what a natural class of related spaces should be. The weak product $\mathcal{H} \odot \mathcal{H}$ and the space of Hankel symbols $\mathcal{X}(\mathcal{H})$ can be associated with a large class of reproducing kernel Hilbert spaces. They may be considered to be the analogs of H^1 and $BMOA$ from the Hardy space theory. In fact, in some generality one shows that $(\mathcal{H} \odot \mathcal{H})^* = \mathcal{X}(\mathcal{H})$, and that Hankel symbols define operators on $\mathcal{H}$ whose null spaces are invariant for all multiplication operators.

In these talks I will present details of this set-up with a view of what they say about the Dirichlet- and Drury-Arveson spaces.